

Nuclear and Particle Physics - Problem Set 2 - Solution

Problem 1)

- a) Here, we are only interested in the neutrons that hit the detector (i.e., that make it into the kinematical “bin” spanned by the detector acceptance). The solid angle covered by the detector is $\Delta\Omega = 10 \text{ cm}^2 / (10\text{m})^2 = 10^{-5} \text{ sterad}$. The reaction is supposed to be isotropic, $d\sigma/d\Omega = 40 \text{ mb/sr}$, and hence the partial cross section for scattering into this $\Delta\Omega$ is $\Delta\sigma = 400 \text{ nb (nano-barn)} = 4 \cdot 10^{-31} \text{ cm}^2$. The luminosity is $L = 10^{11} / \text{s} \cdot 10 \text{ cm} \cdot 0.0708 \text{ g/cm}^3 \cdot 6.022 \cdot 10^{23} \text{ H/g} = 4.264 \cdot 10^{34} \text{ cm}^{-2}\text{s}^{-1}$ (1 g of hydrogen is roughly one mol), and the reaction rate is therefore 17,056 Hz (quite a lot).
- b) The density of carbon is 2.265 g/cm^3 , and the atomic weight (by definition) is exactly 12, so the areal density is $n = 2.265 \text{ g/cm}^3 \cdot 5\text{cm} \cdot 1\text{mol}/12\text{g} \cdot 6.022 \cdot 10^{23} / \text{mol} = 5.685 \cdot 10^{23} / \text{cm}^2$. To stay below the stated luminosity, we need less than $10^{34} / 5.685 \cdot 10^{23} \text{ electrons/s} = 1.76 \cdot 10^{10} \text{ electrons/s}$. Each electron has a charge of $1.602 \cdot 10^{-19} \text{ C}$, so this corresponds to a current of 2.8 nA (pretty low compared to the 100 μA that the accelerator can deliver). CLAS makes up for the low luminosity by its large solid angle (approximately 2π as opposed to a few msr for most spectrometers).

Problem 2)

- a) Let's call the incoming electron momentum vector $\vec{k}$ and the outgoing vector $\vec{k}'$ (both with the same magnitude k). Then the momentum transfer vector is $\vec{q} = \vec{k} - \vec{k}'$ and its magnitude squared is

$$\vec{q}^2 = (\vec{k} - \vec{k}')^2 = k^2 + k'^2 - 2\vec{k} \cdot \vec{k}' = 2k^2 - 2k^2 \cos \theta_e = 4k^2 \sin^2(\theta_e/2).$$

Hence, the magnitude of the momentum transferred is $2k \sin \theta_e/2$.

- b) With the given assumptions, the square of the 3-vector part of the 4-momentum transfer is

$$\vec{q}^2 = (\vec{k} - \vec{k}')^2 = \vec{k}^2 + \vec{k}'^2 - 2\vec{k} \cdot \vec{k}' = E^2 + E'^2 - 2EE' \cos \theta_e$$

In anticipation of part c), we can also write this as

$$E^2 + E'^2 - 2EE' \cos \theta_e = (E - E')^2 + 2EE' - 2EE' \cos \theta_e = (E - E')^2 + 4EE' \sin^2(\theta_e/2).$$

The magnitude of q is then just the square root of this expression.

Note that for the case $E = E'$, this result agrees with the one above!

- c) $Q^2 := -q^\mu q_\mu = \vec{q}^2 - q^{02} = (E - E')^2 + 4EE' \sin^2(\theta_e/2) - (E - E')^2 = 4EE' \sin^2(\theta_e/2)$. Again, notice how similar this answer is to a), although here it is a relativistic invariant.